



ÉCOLE POLYTECHNIQUE  
FÉDÉRALE DE LAUSANNE

# MOSES Workshop

*Modelling and Optimization of Ship Energy Systems*

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## Static and Dynamic Optimization of Synthesis, Design and Operation of Total Energy Systems of Ships

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# Contents

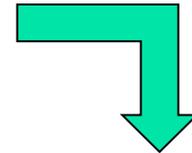
- 1. Introduction: Concepts and Definitions**
  - 2. Levels of Energy Systems Optimization**
  - 3. Formulation and Solution Methods of the Static Optimization Problem**
  - 4. Formulation and Solution Methods of the Dynamic Optimization Problem**
  - 5. Application Examples**
  - 6. Closure**
- Bibliography**

# 1. Introduction: Concepts and Definitions

**Conventional design procedure: *workable system*:**

**A system that delivers the required energy products under certain constraints.**

**Scarcity of physical and economic resources, and deterioration of the environment**



**Need of the *best system*, obtained by formal optimization procedure.**

## ***Why Optimization?***

***Because the goal is not the last,  
but the best.***

**Aristotle (384-322 B.C.)**

***Second Book of Physics***

## 1. Introduction

### *Definition of optimization:*

**Optimization is the act of obtaining the best result under given circumstances or, expressed more formally, the process of finding the conditions that give the maximum or minimum of a function (**objective function**).**

## 1. Introduction

### *Intertemporal optimization:*

**The optimization that takes into consideration the various operating conditions that a system encounters or is expected to encounter throughout its life time and determines the mode of operation at each instant of time that results in the overall minimum or maximum of the general objective function.**



## 1. Introduction

### Dependence on time

```
graph TD; A[Dependence on time] --> B[Discrete]; A --> C[Continuous];
```

**Discrete**

**Solution:**  
a sequence of optimal  
decisions in discrete time  
over the planning period  
or planning horizon.

**Continuous**

**Solution:**  
a time path or curve of  
optimal decisions in  
continuous time over the  
planning period or  
planning horizon.

## 1. Introduction

### *Intertemporal static (or pseudo-dynamic) optimization:*

**Example: Operation optimization of an energy system under time-varying conditions, if the period of operation can be decomposed in a series of time intervals with steady-state operation in each interval, independent of each other.**

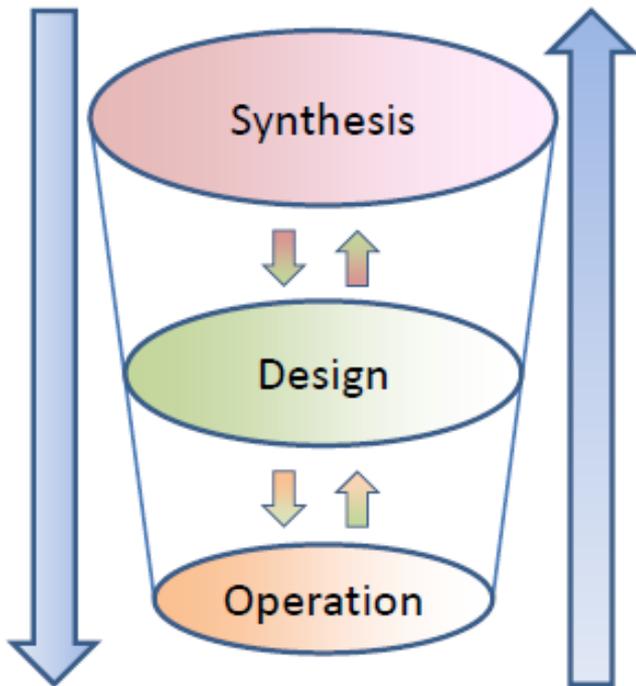
**The initial problem is transformed into a series of static optimization problems.**

## 1. Introduction

*Intertemporal dynamic optimization:*

**Direct or indirect interdependency  
among the modes of operation.**

## 2. Levels of Energy Systems Optimization



- A. Synthesis:** components and their interconnections.
- B. Design:** technical characteristics of components and properties of substances at the nominal (design) point.
- C. Operation:** operating properties of components and substances.

*Figure 2.1: The three inter-related levels of optimization.*

## 2. Levels of Energy Systems Optimization

**The complete optimization problem stated as a question:**

*What is the synthesis of the system, the design characteristics of the components and the operating strategy that lead to an overall optimum?*

# **3. Formulation and Solution Methods of the Static Optimization Problem**

### 3. Formulation and Solution Methods of the Static Optimization Problem

#### 3.1 Mathematical Statement of the Static Optimization Problem

##### Mathematical formulation of the optimization problem

$$\underset{\mathbf{x}}{\text{minimize}} \quad f(\mathbf{x}) \quad (1)$$

with respect to:

$$\mathbf{x} = (x_1, x_2, \dots, x_n) \quad (2)$$

subject to the constraints:

$$h_i(\mathbf{x}) = 0 \quad i = 1, 2, \dots, m \quad (3)$$

$$g_j(\mathbf{x}) \leq 0 \quad j = 1, 2, \dots, p \quad (4)$$

$\mathbf{x}$             **set of independent variables,**  
 $f(\mathbf{x})$         **objective function,**  
 $h_i(\mathbf{x})$        **equality constraint functions,**  
 $g_j(\mathbf{x})$        **inequality constraint functions.**

### 3.1 Mathematical Statement of the Static Optimization Problem

Alternative expression:

$$\min_{\mathbf{v}, \mathbf{w}, \mathbf{z}} f(\mathbf{v}, \mathbf{w}, \mathbf{z}) \quad (1)'$$

- v** set of independent variables for operation optimization,
- w** set of independent variables for design optimization,
- z** set of independent variables for synthesis optimization.

$$\mathbf{x} \equiv (\mathbf{v}, \mathbf{w}, \mathbf{z}) \quad (5)$$

**Design optimization:**  $\min_{\mathbf{v}, \mathbf{w}} f_d(\mathbf{v}, \mathbf{w}) \quad (1)_d$

**Operation optimization:**  $\min_{\mathbf{v}} f_{op}(\mathbf{v}) \quad (1)_{op}$

### 3.1 Mathematical Statement of the Static Optimization Problem

Maximization is also covered by the preceding formulation, since:

$$\min_{\mathbf{x}} f(\mathbf{x}) = \max_{\mathbf{x}} \{-f(\mathbf{x})\} \quad (6)$$

## 3. Formulation and Solution Methods of the Static Optimization Problem

### 3.2 Objective Functions

#### Examples of Objective Functions:

- minimization of weight of the system,
- minimization of size of the system,
- maximization of efficiency,
- minimization of fuel consumption,
- minimization of exergy destruction,
- maximization of the net power density,
- minimization of emitted pollutants,
- minimization of life cycle cost (LCC) of the system,
- maximization of the internal rate of return (IRR),
- minimization of the payback period (PBP),
- etc.

## 3.2 Objective Functions

### **Multiobjective optimization:**

**An attempt to take two or more objectives into consideration simultaneously.**

## 3. Formulation and Solution Methods of the Static Optimization Problem

### 3.3 Equality and Inequality Constraints

#### Equality Constraints:

Model of the components and of the system.

#### Inequality Constraints:

Imposed by safety and operability requirements.

Quantities appearing in the equality and inequality constraints:

- *parameters*
- *independent variables*
- *dependent variables*

## 3. Formulation and Solution Methods of the Static Optimization Problem

### 3.4 Parameters and Variables

#### Parameters:

Quantities that keep a constant value during optimization.

#### Independent variables:

Their values do not depend on other variables.

#### Dependent variables:

Their values depend on the values of other variables.

The number of dependent variables is equal to the number of equality constraints.

### 3. Formulation and Solution Methods of the Static Optimization Problem

#### 3.5 Methods for Solution of the Static Optimization Problem

- (i) ***Search methods:*** They calculate the values of the objective function at a number of combinations of values of the independent variables and seek for the optimum point. They do not use derivatives.
- (ii) ***Calculus methods:*** They use first and (some of them) second derivatives; this is why they are called also *gradient methods*.
- (iii) ***Stochastic or Evolutionary methods:*** Methods and algorithms such as Genetic Algorithms (GA), Simulating Annealing (SA), Particle Swarm Optimization (PSO), Neural Networks belong to this category.

### 3.5 Methods for Solution of the Static Optimization Problem

Two of the most successful methods for optimization of energy systems:

- **Generalized Reduced Gradient (GRG)**
- **Sequential Quadratic Programming (SQP).**

Also a **combination** of a stochastic algorithm (e.g. **GA, PSO**) with a deterministic algorithm (e.g. **GRG, SQP**).

## 4. Formulation and Solution Methods of the Dynamic Optimization Problem (DOP)

## 4.1 Mathematical Statement of the DOP

### Objective function (Mayer form):

$$\underset{\mathbf{z}(t), \mathbf{y}(t), \mathbf{u}(t), t_f, \mathbf{w}}{\text{minimize}} J[\mathbf{z}(t_f), \mathbf{y}(t_f), \mathbf{u}(t_f), t_f, \mathbf{w}] \quad (12)$$

**subject to:**  $\mathbf{H}(\dot{\mathbf{z}}(t), \mathbf{z}(t), \mathbf{y}(t), \mathbf{u}(t), t, \mathbf{w}) = 0 \quad (13)$

$$\mathbf{G}(\dot{\mathbf{z}}(t), \mathbf{z}(t), \mathbf{y}(t), \mathbf{u}(t), t, \mathbf{w}) \leq 0 \quad (14)$$

**Initial conditions:**  $\mathbf{z}(0) = \mathbf{z}^0 \quad (15)$

**Point conditions:**  $\mathbf{P}_s(\mathbf{z}(t_s), \mathbf{y}(t_s), \mathbf{u}(t_s), t_s, \mathbf{w}) = 0, \quad (16)$

$$t_s \in [t_0, t_f]$$

**Bounds:**  $\mathbf{z}^L \leq \mathbf{z}(t) \leq \mathbf{z}^U \quad (17a)$

$$\mathbf{y}^L \leq \mathbf{y}(t) \leq \mathbf{y}^U \quad (17b)$$

$$\mathbf{u}^L \leq \mathbf{u}(t) \leq \mathbf{u}^U \quad (17c)$$

$$\mathbf{w}^L \leq \mathbf{w} \leq \mathbf{w}^U \quad (17d)$$

$$t_f^L \leq t_f \leq t_f^U \quad (17e)$$

$J$  scalar objective functional

$\mathbf{H}$  differential-algebraic equality constraints

$\mathbf{G}$  differential-algebraic inequality constraints

$\mathbf{P}_s$  additional point conditions at times  $t_s$  (including  $t_f$ )

$\mathbf{z}$  differential state profile vector

$\mathbf{z}^0$  initial values of  $\mathbf{z}(t)$

$\mathbf{y}$  algebraic state profile vector

$\mathbf{u}$  control (independent variables) profile vector

$\mathbf{w}$  time-independent variable vector

$t_f$  final time

## 4.1 Mathematical Statement of the DOP

**Objective function (Bolza form):**

$$J[\mathbf{z}(t_f), \mathbf{y}(t_f), \mathbf{u}(t_f), t_f, \mathbf{w}] = Q(\mathbf{z}(t_f), \mathbf{y}(t_f), t_f, \mathbf{w}) + \int_{t_0}^{t_f} F(\mathbf{z}(t), \mathbf{y}(t), \mathbf{u}(t), t, \mathbf{w}) dt$$

**(18)**

## 4.1 Mathematical Statement of the DOP

### Statement of the discrete problem

**Objective function (additively separable across time):**

$$\underset{\mathbf{z}, \mathbf{y}, \mathbf{u}, t_f, \mathbf{w}}{\text{minimize}} J[\mathbf{z}, \mathbf{y}, \mathbf{u}, t_f, \mathbf{w}] = Q(\mathbf{z}_N, \mathbf{y}_N, N, \mathbf{w}) + \sum_{n=1}^N F(\mathbf{z}_n, \mathbf{y}_n, \mathbf{u}_n, n, \mathbf{w}) \quad (19)$$

**where  $N$  the number of time intervals:**  $t_f - t_0 = N \cdot \Delta t_n$

**The optimization problem can be solved by optimal control theory or dynamic programming.**

## 4. Formulation and Solution Methods of the DOP

### 4.2 Methods for Solution of the Dynamic Optimization Problem

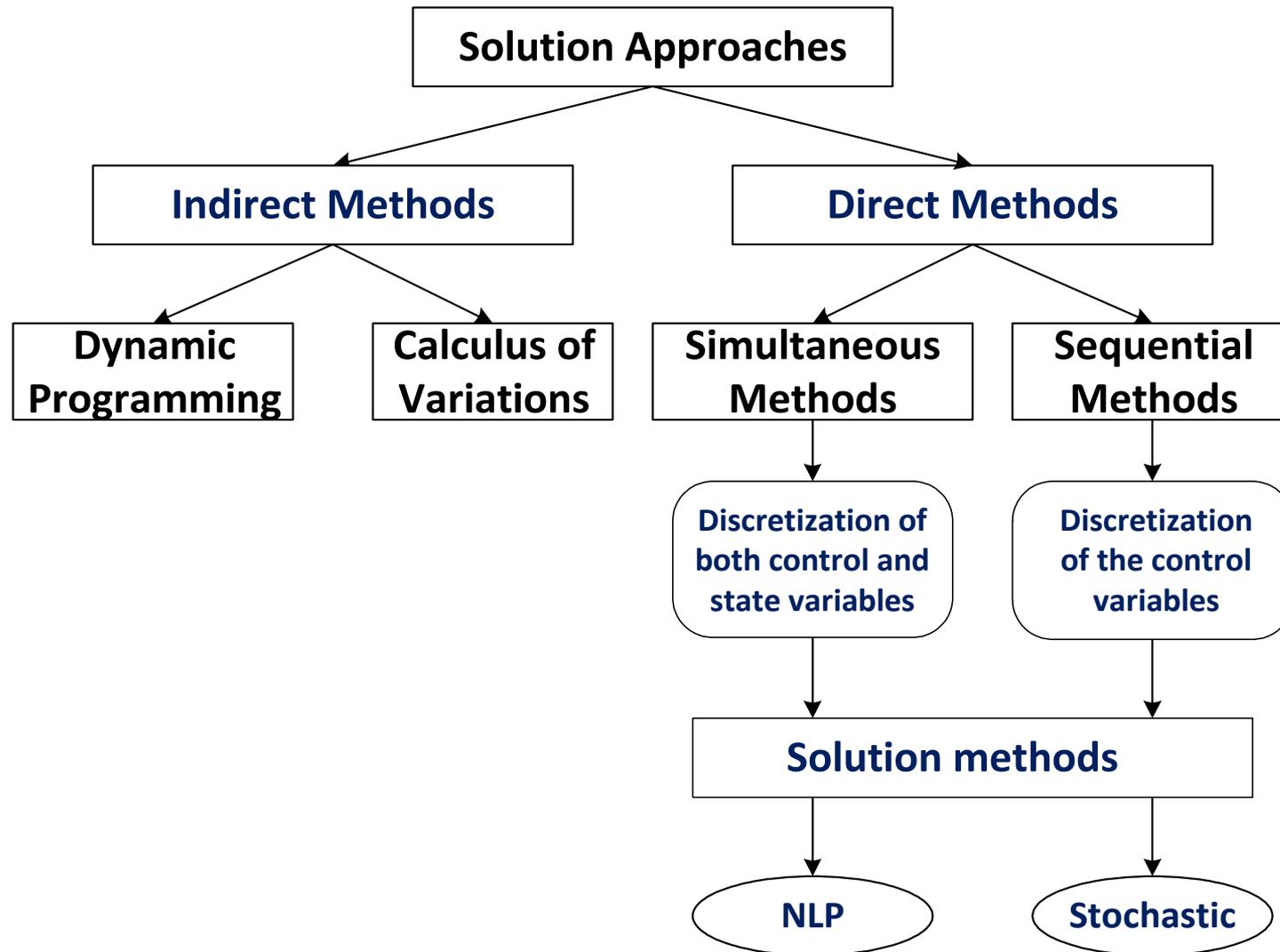


Fig. 4.1 Solution approaches for Dynamic Optimization Problems.

## 4.2.2 Direct Methods

Discretization can be: **Constant**  
**Linear**  
**Polynomial** →

$$u_i(t) = \sum_{k=1}^{N_{col}} \psi_k \left( \frac{t - t_{i-1}}{L_i} \right) u_{i,k}$$

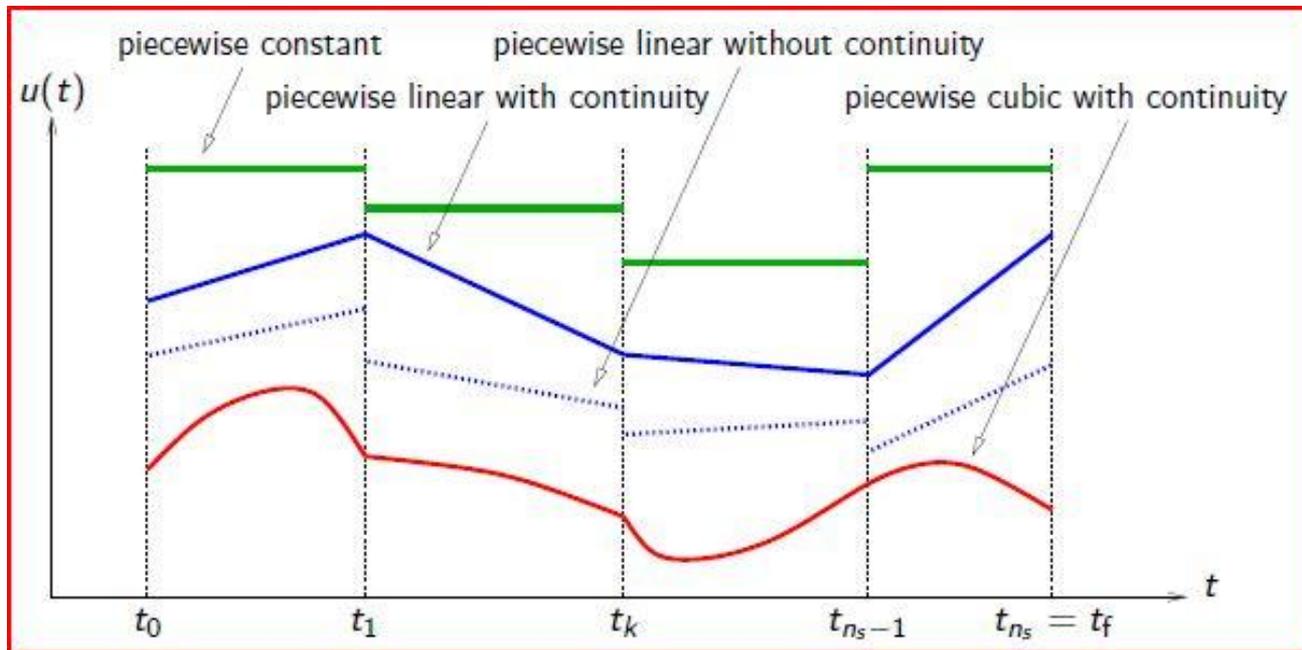


Fig. 4.2 Constant, linear and polynomial approximations.

# **5. Application Examples**

## **5.1 Example 1:**

### **Intertemporal Static Optimization of Synthesis, Design and Operation of a Marine Energy System**

**Work of Ph.D. Candidate George Sakalis**

## 5.2 Example 1

### 5.1.1 Description of the system

The optimal synthesis, design and operation of a system that will cover all energy needs of a ship is requested.

The operation is approximated with three modes of steady state operation (port and idle periods are omitted). Loads are given in Table 1.

*Table 1: Energy profile of the ship for one typical year.*

| Mode | $\dot{W}_{p,y}$ | $\dot{W}_{e,y}$ | $\dot{Q}_{hl,y}$ | $t_y$ |
|------|-----------------|-----------------|------------------|-------|
| y    | kW              | kW              | kW               | hours |
| 1    | 26000           | 1500            | 400              | 2690  |
| 2    | 22000           | 1500            | 300              | 1575  |
| 3    | 14000           | 700             | 200              | 1620  |

# Example 1: Description of the system

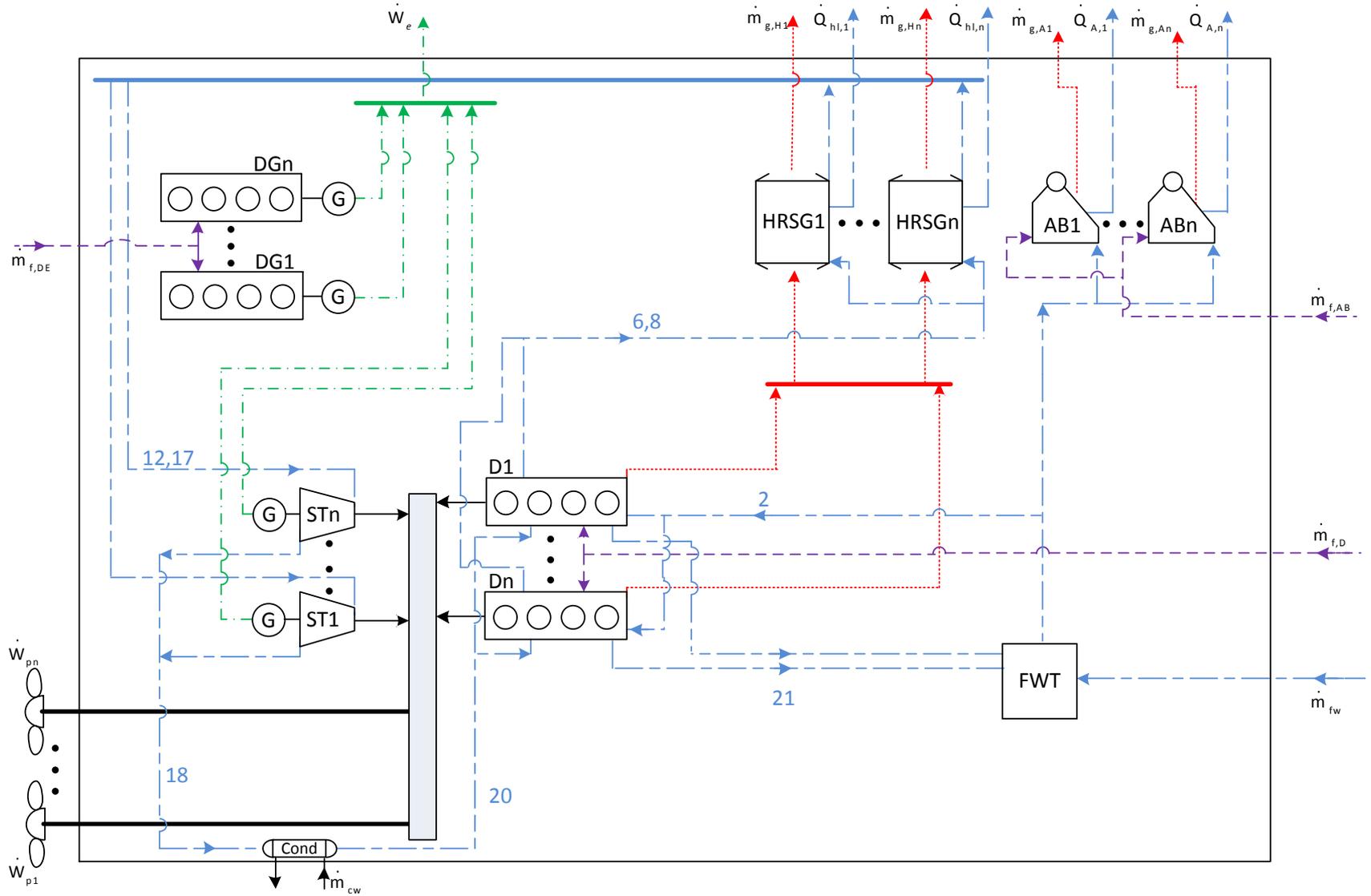


Figure 5.1: Superconfiguration of the integrated system of Example 1.

# Example 1: Description of the system

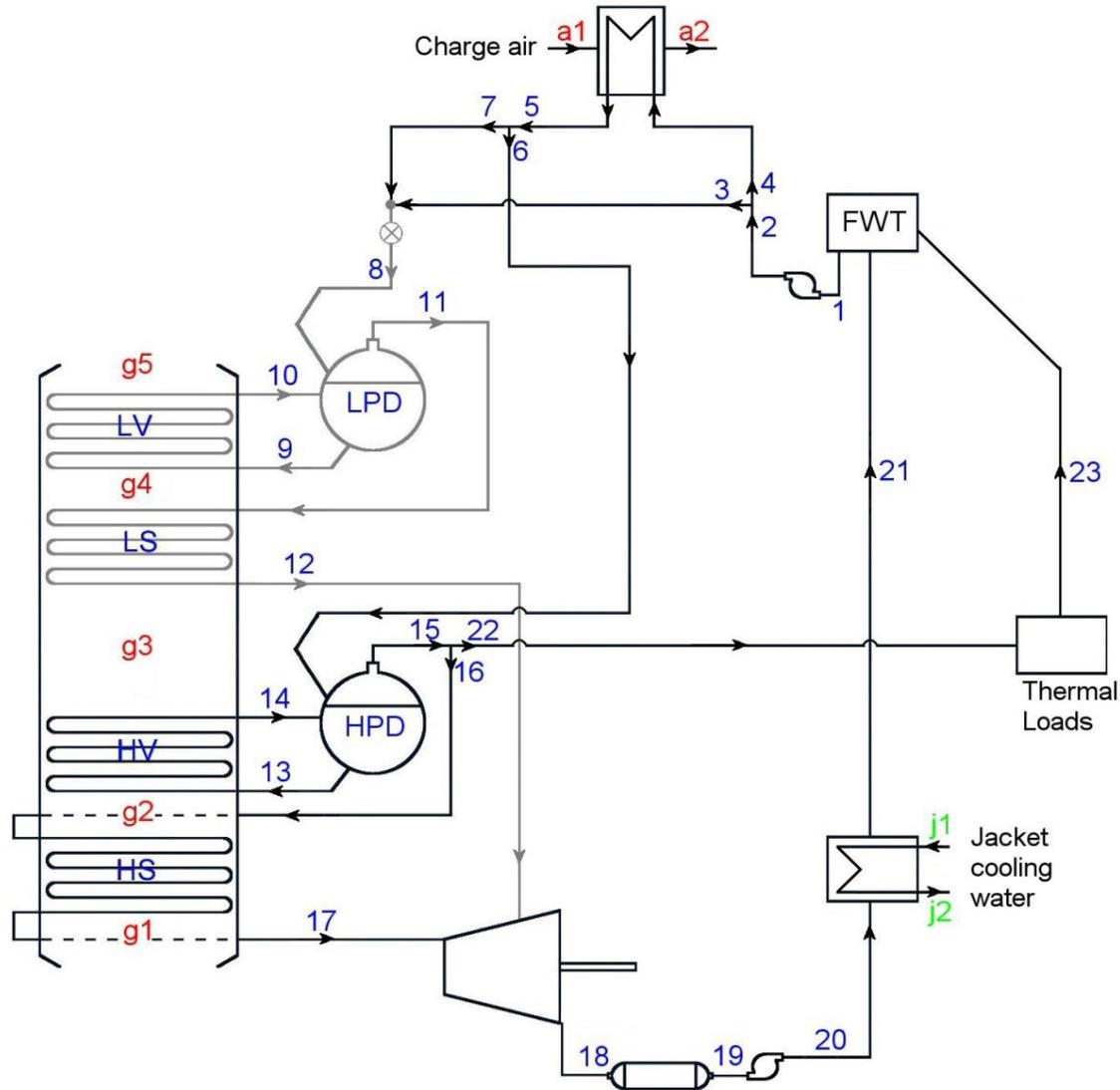


Figure 5.2: Internal structure of the HRSG of Example 1.

## 5.1 Example 1

### 5.1.2 Mathematical Statement of the Optimization Problem

#### Objective function:

$$\begin{aligned}
 \text{mim } PWC = & \sum_{x=1}^{x=n_{D,max}} C_{c,D,x} + \sum_{x=1}^{x=n_{DE,max}} C_{c,DE,x} + \sum_{z=1}^{z=n_{HRSG,max}} C_{c,HRSG,z} + \sum_{v=1}^{v=n_{ST,max}} C_{c,ST,v} + \\
 & + PWF(N_Y, f, i) \times \sum_{y=1}^{N_T} \left\{ \sum_{x=1}^{x=n_{D,max}} \dot{m}_{f,Dxy} t_{Dxy} C_{f,D} + \sum_{x=1}^{x=n_{DE,max}} \dot{m}_{f,DExy} t_{DExy} C_{f,DE} \right\} + \\
 & + PWF(N_Y, f, i) \times \sum_{y=1}^{N_T} \left\{ \sum_{k=D,DE,ST} \left[ \sum_{x=1}^{n_{k,max}} C_{om,kxy} \dot{W}_{kxy} t_{kxy} \right] + \sum_{z=1}^{z=n_{HRSG,max}} C_{om,HRSGz,y} \dot{Q}_{HRSGz,y} t_{HRSGz,y} \right\}
 \end{aligned} \tag{20}$$

First line: Capital cost of equipment

Second line: Cost of fuel

Third line: Operation and maintenance cost (except fuel).

## Example 1: Mathematical Statement of the Optimization Problem

Equality constraints coming from the need to cover the loads:

$$\sum_{x=1}^{x=n_{D,y}} \dot{W}_{D,x,y} + \sum_{v=1}^{v=n_{ST,y}} \dot{W}_{STp,v,y} = \dot{W}_{p,y} \quad \text{for } y = 1, \dots, N_T \quad (21)$$

$$\sum_{x=1}^{x=n_{DE,y}} \dot{W}_{DEx,y} + \sum_{v=1}^{v=n_{ST,y}} \dot{W}_{STev,y} = \dot{W}_{e,y} \quad \text{for } y = 1, \dots, N_T \quad (22)$$

$$\sum_{z=1}^{z=n_{HRSG,y}} \dot{m}_{hl,z,y} (h_{sat,z,i} - h_{return}) + \sum_{u=1}^{u=n_{AB,y}} \dot{Q}_{AB,u,y} = \dot{Q}_{th,y} \quad \text{for } y = 1, \dots, N_T \quad (23)$$

Additional equality constraints are derived by the simulation of the components and the system.

## Example 1: Mathematical Statement of the Optimization Problem

### Design independent variables:

|                                      |  |
|--------------------------------------|--|
| $\dot{W}_{N,x}$                      | nominal power rating of each Diesel engine $x$   |
| $\dot{m}_{g,des,z}$                  | design exhaust gas mass flow rate of each HRSG $z$   |
| $T_{g,des,z}$                        | design exhaust gas temperature of each HRSG $z$  |
| $P_{HP}, P_{LP}$                     | high and low pressure levels of the HRSGs (common for all and constant)  |
| $T_{HP,z}, T_{LP,z}$                 | temperature of steam produced at high and low pressure levels for each HRSG $z$                                  |
| $\dot{m}_{HP,z}, \dot{m}_{LP,z}$     | mass flow rates of steam produced at high and low pressure levels for each HRSG $z$                              |
| $T_{HPST,v}, T_{LPST,v}$             | design temperature of steam at high and low pressure levels for each ST $v$                                      |
| $\dot{m}_{HPST,v}, \dot{m}_{LPST,v}$ | design mass flow rate of steam at high and low pressure levels for each ST $v$                                   |
| $\dot{m}_{HP,z}, \dot{m}_{LP,z}$     | mass flow rates of steam produced at high and low pressure levels for each HRSG $z$                              |
| $\rho_{m,z}$                         | by-pass ratio of feed water from the charge air cooler for each HRSG $z$<br>( $\rho_m = \dot{m}_3 / \dot{m}_2$ ) |
| $\varepsilon_{a,z}$                  | effectiveness of charge air heat exchanger for each HRSG $z$   |
| $T_{a,z}$                            | charge air inlet temperature at charge air heat exchanger for each HRSG $z$ .                                    |
| $\dot{m}_{a,z}$                      | charge air design mass flow rate at charge air heat exchanger for each HRSG $z$ .                                |

## Example 1: Mathematical Statement of the Optimization Problem

### Operation independent variables:

|                   |   |
|-------------------|---|
| $n_{D,y}$         | number of prime movers in operation   |
| $g_{x,y}$         | HRSG ( $z$ number) at which the exhaust gas from prime mover $x$ is delivered |
| $\lambda_{D,y}$   | fraction of the propulsion power covered by the operating prime movers        |
| $\dot{W}_{b,x,y}$ | brake power of operating engine $x$   |
| $\dot{Q}_{H,z,y}$ | thermal load fraction covered by HRSG $z$                                     |

### Examples of inequality constraints:

Nominal power output of Diesel engines:  $\dot{W}_{N,x} \leq 20.000 \text{ kW}$  (24a)

Minimum temperature of exhaust gases:  $T \geq 160^\circ\text{C}$  (24b)

Quality of steam at the exit of steam turbine:  $\chi \geq 0.85$  (24c)

## 5.1 Example 1

### 5.1.3 Solution of the Optimization Problem

**Objective function multimodal with discontinuous first derivatives in the search space.**



**Gradient-based methods inappropriate.**



**Solution by a genetic Algorithm.**

## Example 1: Solution of the Optimization Problem

*Table 2: Economic parameters for Example 1.*

| Parameter                    | Value     |
|------------------------------|-----------|
| Lifecycle of the ship, $N_y$ | 20 years  |
| Inflation rate, $f$          | 3%        |
| Interest rate, $i$           | 8%        |
| Fuel price, $c_f$            | 600 €/ton |

## Example 1: Solution of the Optimization Problem

*Table 3a: Optimal synthesis of the system.*

|  |          |
|--|----------|
| <b>Number of Diesel engines (prime movers)</b> | <b>2</b> |
| <b>Number of HRSGs</b>                         | <b>2</b> |
| <b>Number of steam turbines</b>                | <b>1</b> |

## Example 1: Solution of the Optimization Problem

*Table 3b: Optimal design specifications of the system components.*

| Variable                            |        | Engine 1 | Engine 2 |
|-------------------------------------|--------|----------|----------|
| Main engine nominal power (MCR)     | (kW)   | 14641    | 15989    |
| Heat recovery steam generator       |        | HRSG 1   | HRSG 2   |
| Thermal power                       | (kW)   | 5386     | 4855     |
| Exhaust gas mass flow rate          | (kg/s) | 27.13    | 29.66    |
| Inlet exhaust gas temperature       | (°C)   | 349.88   | 321.73   |
| Outlet exhaust gas temperature      | (°C)   | 171.20   | 173.40   |
| High pressure (HP)                  | (bar)  | 9.008    | 9.008    |
| Low pressure (LP)                   | (bar)  | 4.540    | 4.540    |
| Temperature of HP superheated steam | (°C)   | 318.39   | 287.76   |
| Temperature of LP superheated steam | (°C)   | 162.06   | 164.97   |
| HP steam flow rate                  | (kg/s) | 1.774    | 1.626    |
| LP steam flow rate                  | (kg/s) | 0.335    | 0.351    |
| Steam turbine                       |        |          |          |
| Nominal power                       | (kW)   | 2293.72  |          |
| HP steam flow rate                  | (kg/s) | 2.692    |          |
| LP steam flow rate                  | (kg/s) | 0.591    |          |
| Temperature of HP superheated steam | (°C)   | 307.87   |          |
| Temperature of LP superheated steam | (°C)   | 161.28   |          |
| Rotational speed                    | (RPM)  | 3000     |          |

## Example 1: Solution of the Optimization Problem

*Table 3c: Optimal operating properties.*

| Mode of operation:                     | 1        | 2       | 3        | 1        | 2        | 3 |
|--|----------|---------|----------|----------|----------|---|
| <b>Main engine:</b>                    | 1        |         |          | 2        |          |   |
| <b>Brake power (kW)</b>                | 12141.82 | 6495.93 | 13596.51 | 13163.88 | 14855.84 | 0 |
| <b>Exhaust gas temperature (°C)</b>    | 295.21   | 348.14  | 299.4    | 295.74   | 299.85   | – |
| <b>Ex. gas mass flow rate (kg/s)</b>   | 27.33    | 16.33   | 29.25    | 30.07    | 32.36    | 0 |
| <b>HRSG:</b>                           | 1        |         |          | 2        |          |   |
| <b>Thermal power (kW)</b>              | 3821.22  | 3289.11 | 4196.57  | 4112.18  | 4534.5   | 0 |
| <b>Inlet ex. gas temperature (°C)</b>  | 295.21   | 348.14  | 299.40   | 295.74   | 299.85   | – |
| <b>Outlet ex. gas temperature (°C)</b> | 168.51   | 165.68  | 169.40   | 171.80   | 172.85   | – |
| <b>Steam turbine:</b>                  |          |         |          |          |          |   |
| <b>Power (kW)</b>                      | 2196.13  | 2154.7  | 1107.17  |          |          |   |

## Example 1: Solution of the Optimization Problem

*Table 4: Cost items (costs in €).*

|  |                    |
|--|--------------------|
| <b>Capital cost</b>                                    | 14,279,129         |
| <b>Present worth cost of fuel</b>                      | 178,122,833        |
| <b>Present worth cost of operation and maintenance</b> | 12,140,977         |
| <b>Total PWC (objective function)</b>                  | <b>204,542,939</b> |

### 5.1.4 Comments on the Results

- The optimum system consists of two Diesel engines, two HRSGs and one steam turbine.
- The two Diesel engines do not have the same nominal power output.
- All the electrical load is covered by the steam turbine, which contributes also to the propulsion.
- The HRSGs cover the whole thermal load.
- If no bottoming cycle were installed and waste heat of the main engines were utilized for covering the thermal loads only, while Diesel-generator sets were used for the electrical loads, the PWC would increase by 6.58%.

## **5.2 Example 2:**

# **Intertemporal Dynamic Optimization of Synthesis, Design and Operation of a Marine Energy System**

**Work of Ph.D. Candidate George Tzortzis**

## 5.2 Example 2

### 5.2.1 Description of the system

- The optimal configuration (synthesis), design specifications and operating conditions of an energy system that will cover all energy needs of a ship are requested.
- The ship encounters varying weather conditions along the route and the optimal speed is requested.
- The propulsion power is not known in advance, but it is calculated as a function of speed and weather conditions.
- Since the trip duration is fixed (Table 5), the speed at a certain instant of time affects the speed in other instants, thus a dynamic optimization problem is at hand.

## Example 2: Description of the system

- The distance between ports A and B is  $d_{AB} = 460$  km, while the time schedule of the ship is given in Table 5.
- The electrical and thermal loads are given in Table 6. They are considered constant and known in port, but they are calculated as functions of the brake power of the engine(s) during the trip.
- For simplicity, the weather conditions are described solely by the wind speed, which is depicted as function of space and time for the two directions of the trip in Figure 4, with the help of 3-D plots (contours).

## Example 2: Description of the system

*Table 5: Time schedule of the ship.*

| Mode | Description                        | Duration             |
|------|------------------------------------|----------------------|
| 1    | Loading at port A                  | $t_1 = 9 \text{ h}$  |
| 2    | Loaded trip from port A to port B  | $t_2 = 15 \text{ h}$ |
| 3    | Off-loading at port B              | $t_3 = 9 \text{ h}$  |
| 4    | Ballast trip from port B to port A | $t_4 = 15 \text{ h}$ |
|      | Total round trip                   | $t_f = 48 \text{ h}$ |

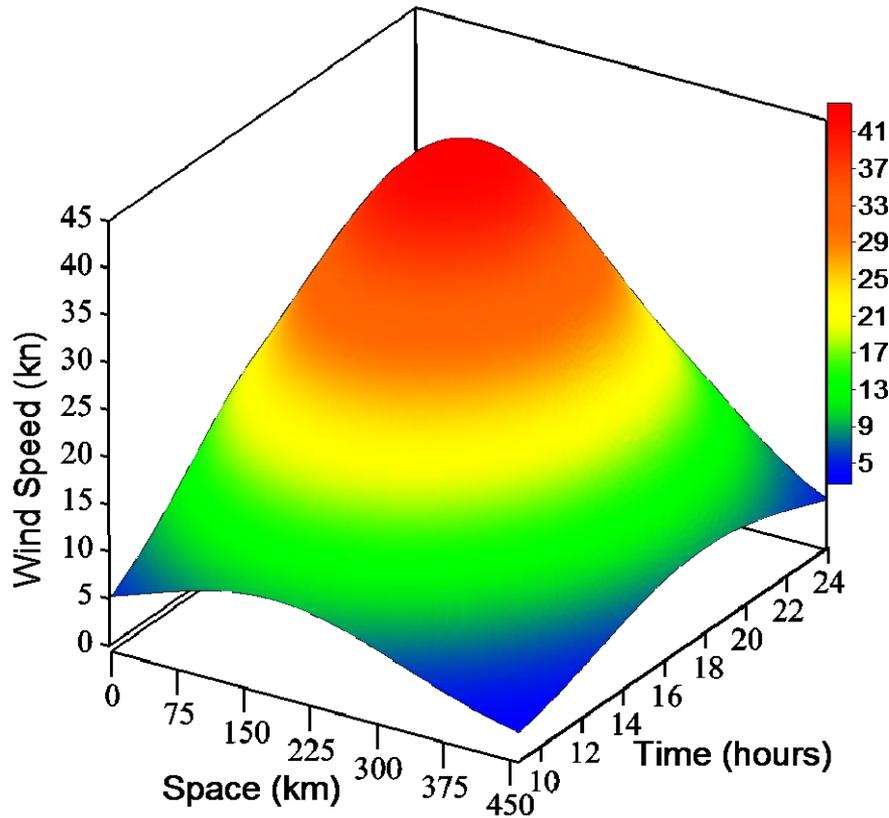
## Example 2: Description of the system

*Table 6: Electrical and thermal loads.*

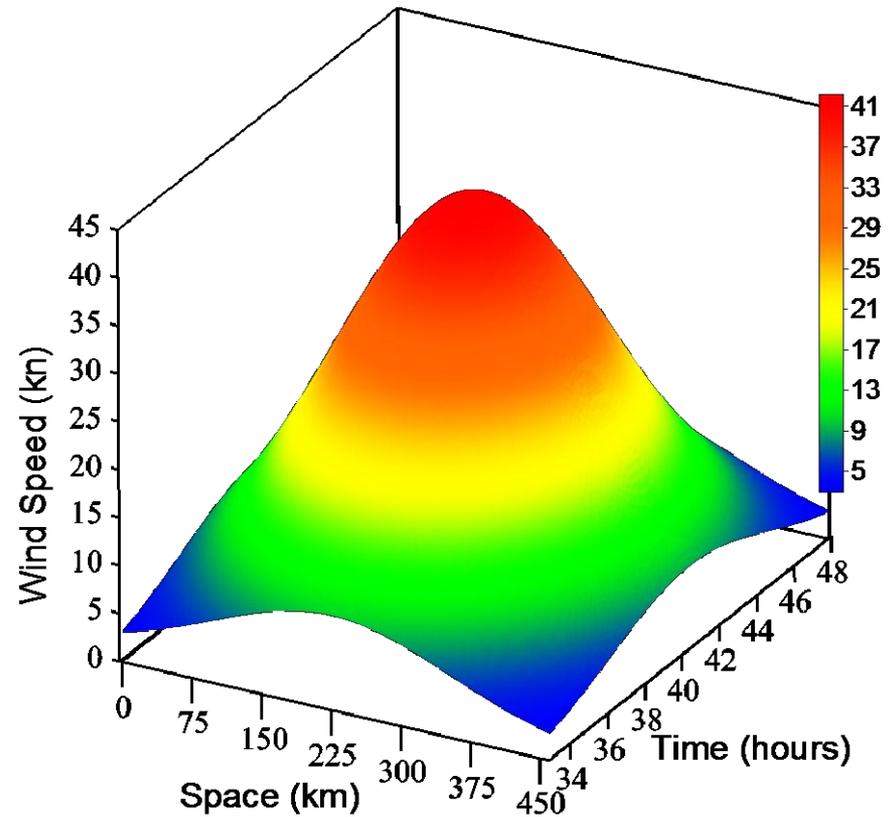
$\dot{W}_b$  : *brake power of main engines*

| Mode | Electrical load (kW)               | Thermal load (kW)                                       |
|------|------------------------------------|---|
| 1    | 1000                               | 800   |
| 2    | $-3451 + 540 \cdot \ln(\dot{W}_b)$ | $150 + \exp(5.37 + 3.93 \cdot 10^{-5} \cdot \dot{W}_b)$ |
| 3    | 3000                               | 4000  |
| 4    | $-3423 + 539 \cdot \ln(\dot{W}_b)$ | $150 + \exp(5.39 + 3.73 \cdot 10^{-5} \cdot \dot{W}_b)$ |

## Example 2: Description of the system



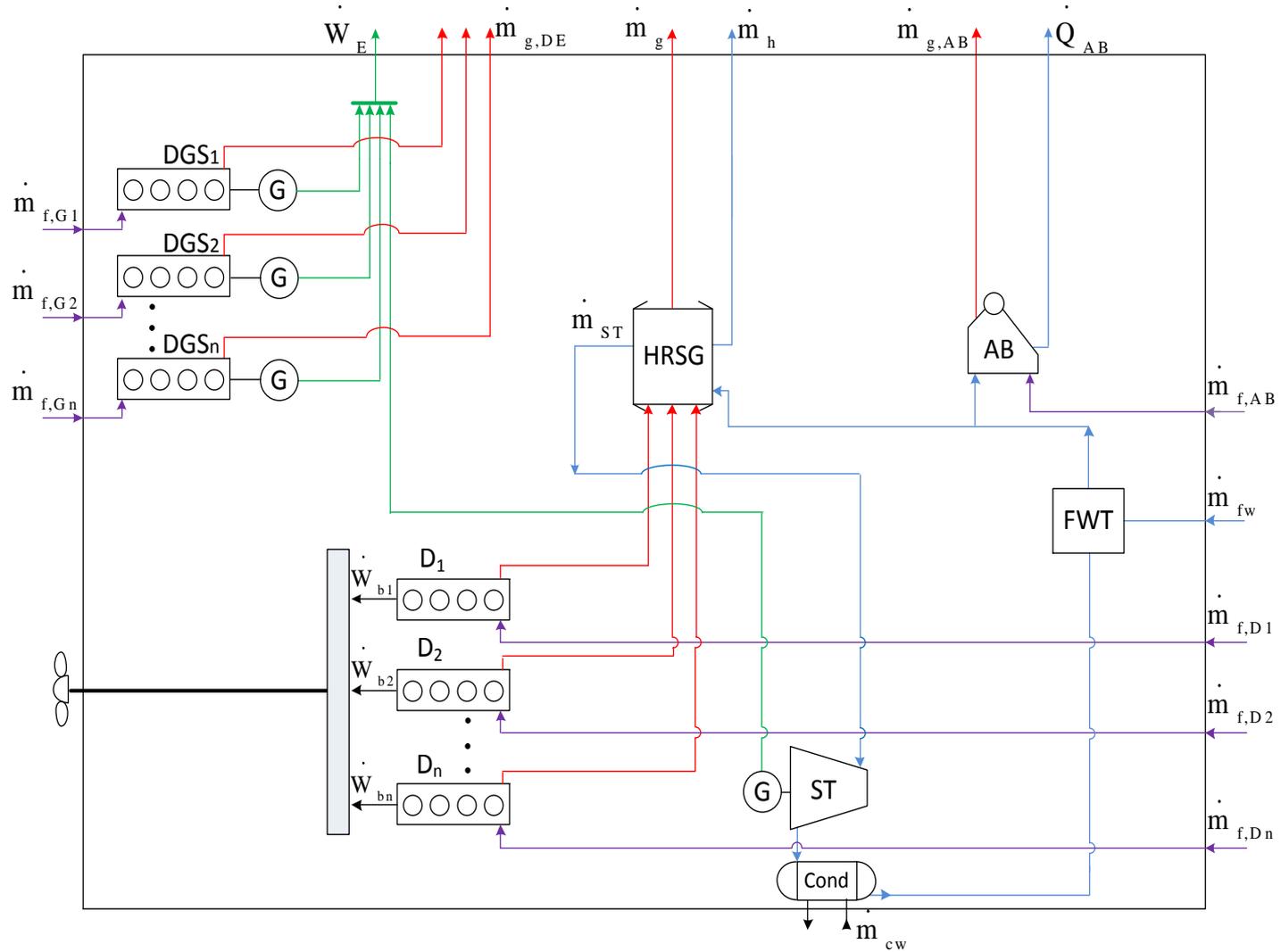
*(a) Loaded trip.*



*(b) Ballast trip.*

*Figure 5.3: Wind speed as a function of space and time.*

## Example 2: Description of the system



**Figure 5.4: Superconfiguration of the energy system of Example 2.**

## 5.2 Example 2

### 5.2.2 Mathematical Statement of the Optimization Problem

Objective function similar to Eq. (20), written here in abbreviated form:

$$\min_{\mathbf{x}} PWC = PWC_c + PWC_f + PWC_{om} \quad (25)$$

Vector of control variables:  $\mathbf{x} = (\mathbf{v}, \mathbf{w}, \mathbf{z})$  (26a)

Synthesis:  $\mathbf{z} = (y_b, y_g, y_{HRSG}, y_{ST})$  (26b)

Design:  $\mathbf{w} = (\dot{W}_{bn_j}, \dot{m}_{g_n}, T_{g_n}, \dot{m}_{ST_n}, \dot{W}_{G_{ni}}, \dot{Q}_{Bn})$  (26c)

Operation:  $\mathbf{v} = (\dot{W}_{b_j}, \lambda_h)$  (26d)

## Example 2: Mathematical Statement of the Optimization Problem

|                           |   |
|---------------------------|---|
| $y_b$                     | number of Diesel engines (integer variable)   |
| $y_g$                     | number of Diesel-generator sets (integer variable)  |
| $y_{HRSG}$                | variable determining the existence of HRSG unit (binary)  |
| $y_{ST}$                  | variable determining the existence of STG unit (binary)   |
| $\dot{W}_{bnj}$           | nominal brake power output of engine $j$ (invariant)  |
| $\dot{m}_{\varepsilon n}$ | nominal exhaust gas mass flow rate for HRSG (invariant)   |
| $T_{\varepsilon n}$       | nominal exhaust gas temperature for HRSG (invariant)  |
| $\dot{m}_{STn}$           | nominal steam mass flow rate for ST (invariant)   |
| $\dot{W}_{Gni}$           | nominal power output of generator set $i$ (invariant)   |
| $\dot{Q}_{Bn}$            | nominal thermal power output of auxiliary boiler (invariant)  |
| $\dot{W}_{bj}$            | brake power of Diesel engine $j$  |
| $\lambda_h$               | fraction of HRSG steam mass flow rate delivered to thermal loads:<br>$\dot{m}_{s,h} = \lambda_h \cdot \dot{m}_s \quad (27)$ |
| $\dot{m}_{s,h}$           | steam mass flow rate from HRSG for covering thermal loads   |
| $\dot{m}_s$               | steam mass flow rate of HRSG unit.  |

## Example 2: Mathematical Statement of the Optimization Problem

**Equality constraints coming from the need to cover the loads:**

$$\sum_j \dot{W}_{b_j} = \dot{W}_b \quad (28)$$

$$\dot{W}_{STG} + \sum_i \dot{W}_{G_i} = \dot{W}_e \quad (29)$$

$$\dot{Q}_h + \dot{Q}_{AB} = \dot{Q} \quad (30)$$

**Additional equality constraints are derived by the simulation of the components and the system.**

**The inequalities of Eq. (24) are valid also in this example.**

## Example 2: Mathematical Statement of the Optimization Problem

Total required brake power of the main engine(s):

$$\dot{W}_b = \frac{V \cdot R_T(V, WS, \mathbf{p})}{\eta_{tot}} \quad (31)$$

where

$V$  speed of the vessel

$R_T$  total resistance of the ship

$WS$  weather state

$\mathbf{p}$  vector of the time independent characteristics of the ship (e.g. dimensions, block coefficient, etc.)

$\eta_{tot}$  total propulsive efficiency

## 5.2 Example 2

### 5.2.3 Solution Procedure of the Optimization Problem

Mixed integer, non-linear dynamic optimization problem.

- Control Vector Parametrization (CVP) approach.
- Single shooting optimization algorithm.
- Software: **gPROMS<sup>®</sup>** via the solver CVP\_SS, which controls the parametrization of the control variables and applies the single shooting algorithm by using the NLPSQP solver.
- The DASOLV solver handles the DAE problem and the computation of sensitivities, while the BDNLSOL is used as the initialization and reinitialisation solver.
- The DASOLV is used for simulation activities.
- The mixed integer part of the problem is handled via the OAERAP solver.

### 5.2.4 Numerical Results

*Table 7: Parameters for the numerical solution.*

| Parameter                          | Value     |
|------------------------------------|-----------|
| Length of time intervals (trips)   | 1 h       |
| Length of time intervals (ports)   | 9 h       |
| Number of time intervals used      | 32        |
| Optimization convergence tolerance | $10^{-7}$ |

## Example 2: Numerical Results

*Table 8: Economic parameters.*

| Parameter                    | Value     |
|------------------------------|-----------|
| Lifecycle of the ship, $N_y$ | 20 years  |
| Interest rate, $i$           | 10%       |
| Fuel price, $c_f$            | 605 €/ton |
| Number of trips per year     | 125       |

## Example 2: Numerical Results

*Table 9a: Optimal synthesis of the system.*

|  |          |
|--|----------|
| <b>Number of Diesel engines (prime movers)</b> | <b>2</b> |
| <b>Number of HRSGs</b>                         | <b>1</b> |
| <b>Number of steam turbines</b>                | <b>1</b> |
| <b>Number of Diesel-generator sets</b>         | <b>2</b> |
| <b>Number of auxiliary boilers</b>             | <b>1</b> |

## Example 2: Numerical Results

*Table 9b: Optimal design specifications of the system components.*

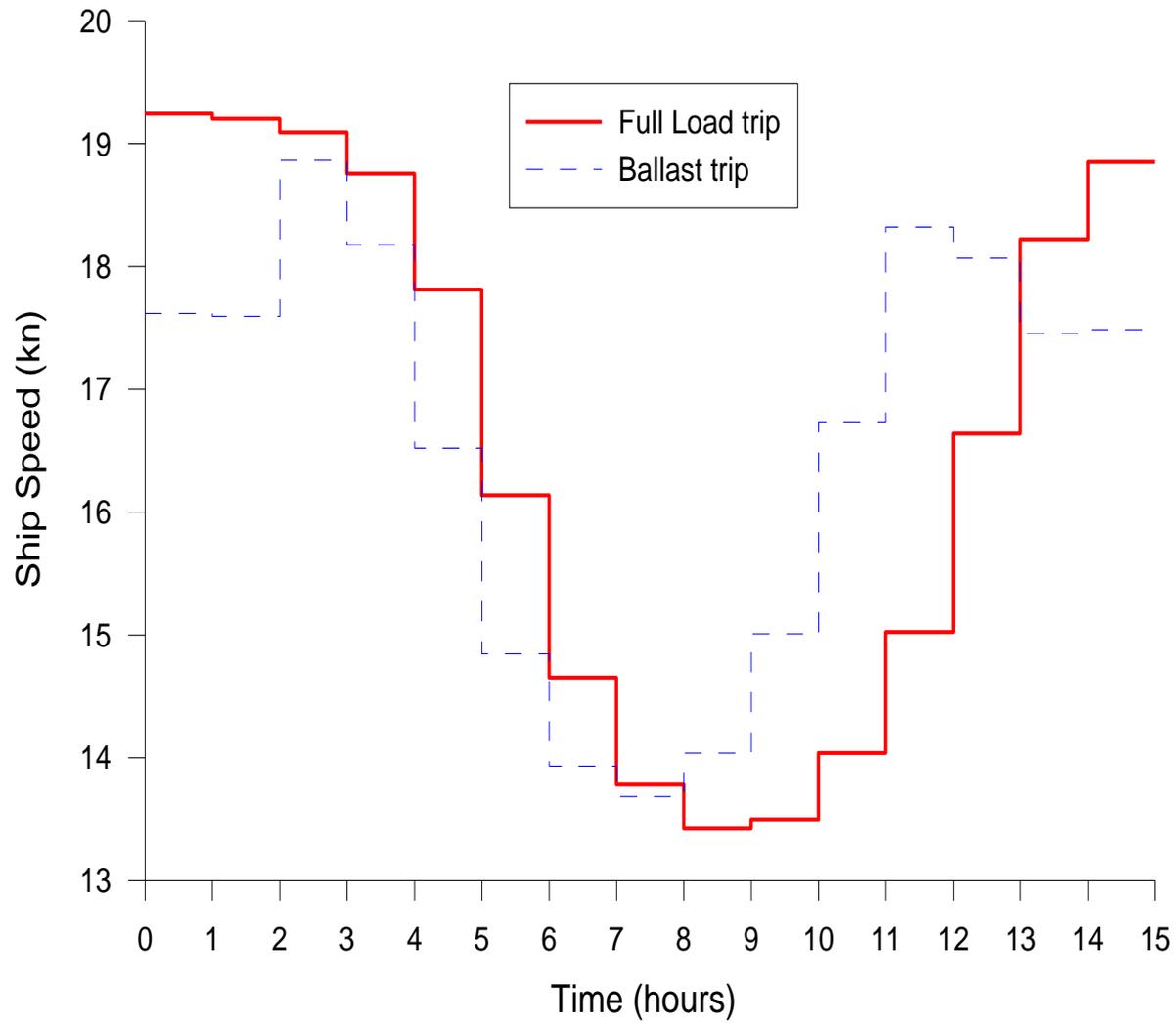
| Variable                                 |        | Engine 1 | Engine 2 |
|--|--------|----------|----------|
| Main engine nominal brake power          | (kW)   | 14840    | 6150     |
| Diesel-generators nominal electric power | (kW)   | 715      | 2402     |
| Heat recovery steam generator            |        |          |          |
| Thermal power                            | (kW)   | 5490     |          |
| Exhaust gas mass flow rate               | (kg/s) | 33.95    |          |
| Nominal inlet exhaust gas temperature    | (°C)   | 294      |          |
| Auxiliary boiler nominal thermal power   | (kW)   | 4000     |          |
| Steam-turbine generator                  |        |          |          |
| Nominal electric power                   | (kW)   | 1560     |          |
| Nominal steam mass flow rate             | (kg/s) | 2.21     |          |

## Example 2: Numerical Results

*Table 10: Cost items (costs in €).*

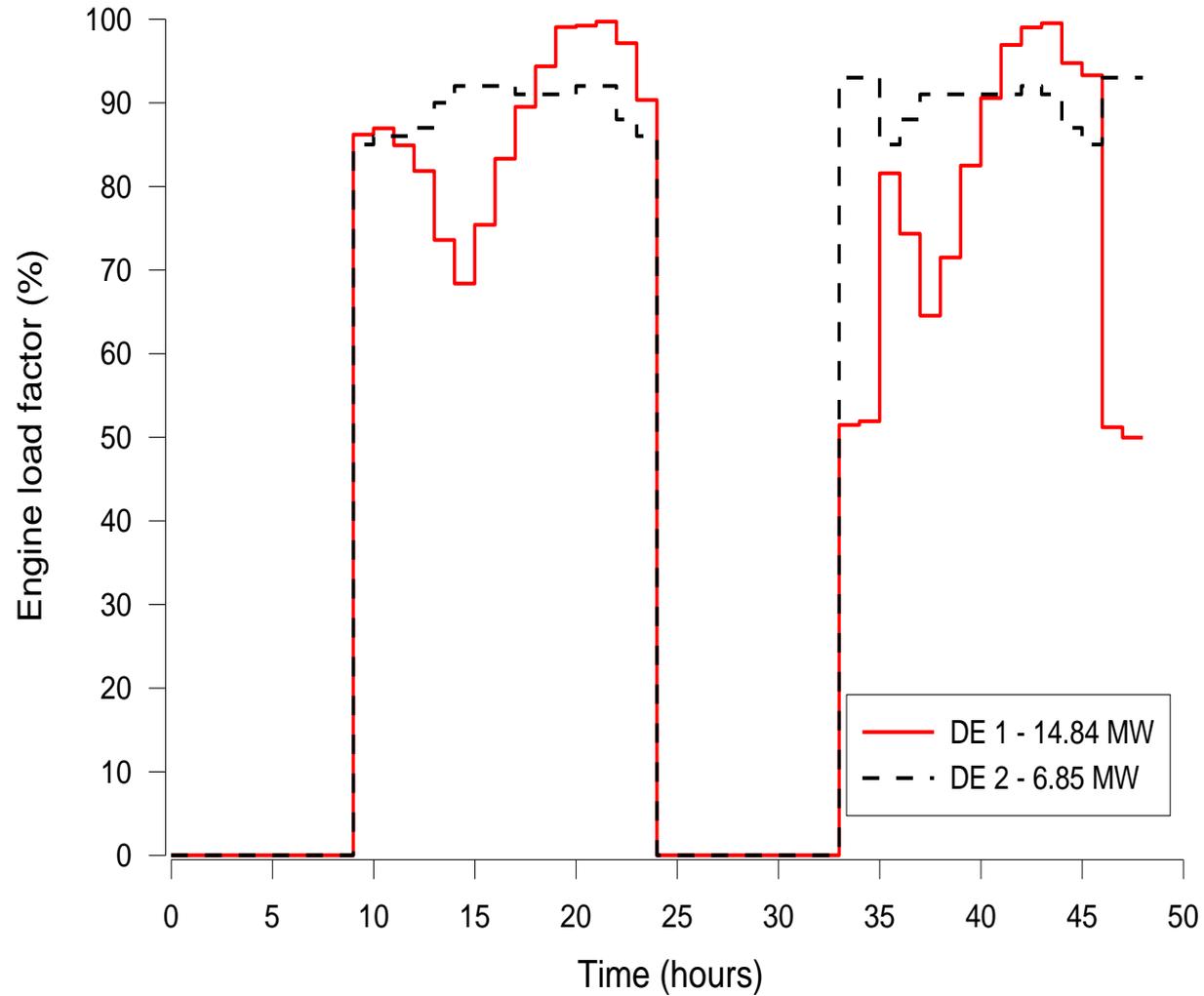
|  |                   |
|--|-------------------|
| <b>Capital cost</b>                                    | 12,930,960        |
| <b>Present worth cost of fuel</b>                      | 77,297,380        |
| <b>Present worth cost of operation and maintenance</b> | 4,873,265         |
| <b>Total PWC (objective function)</b>                  | <b>95,101,605</b> |

## Example 2: Numerical Results



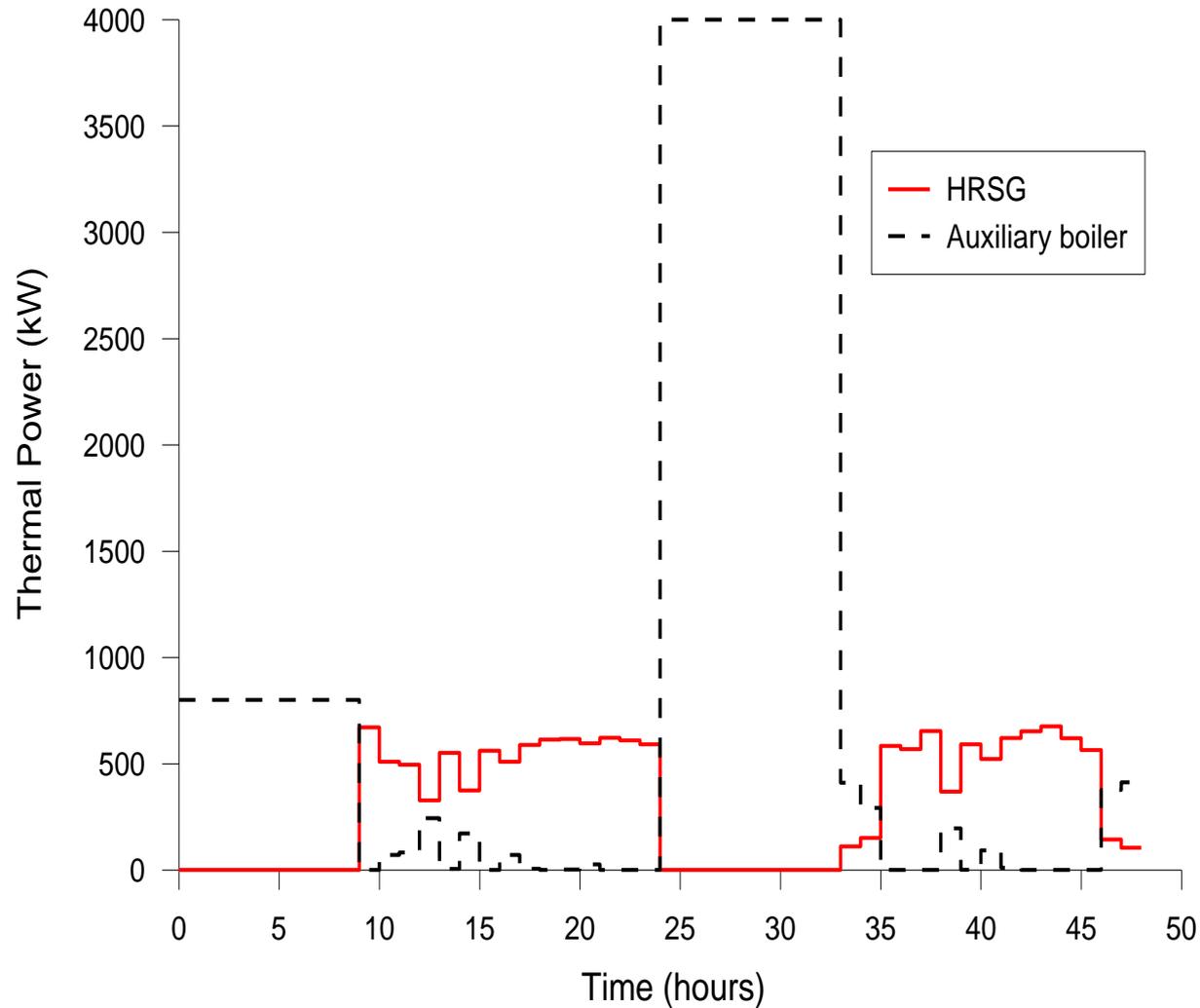
*Figure 5.5: Optimal ship speed versus time.*

## Example 2: Numerical Results



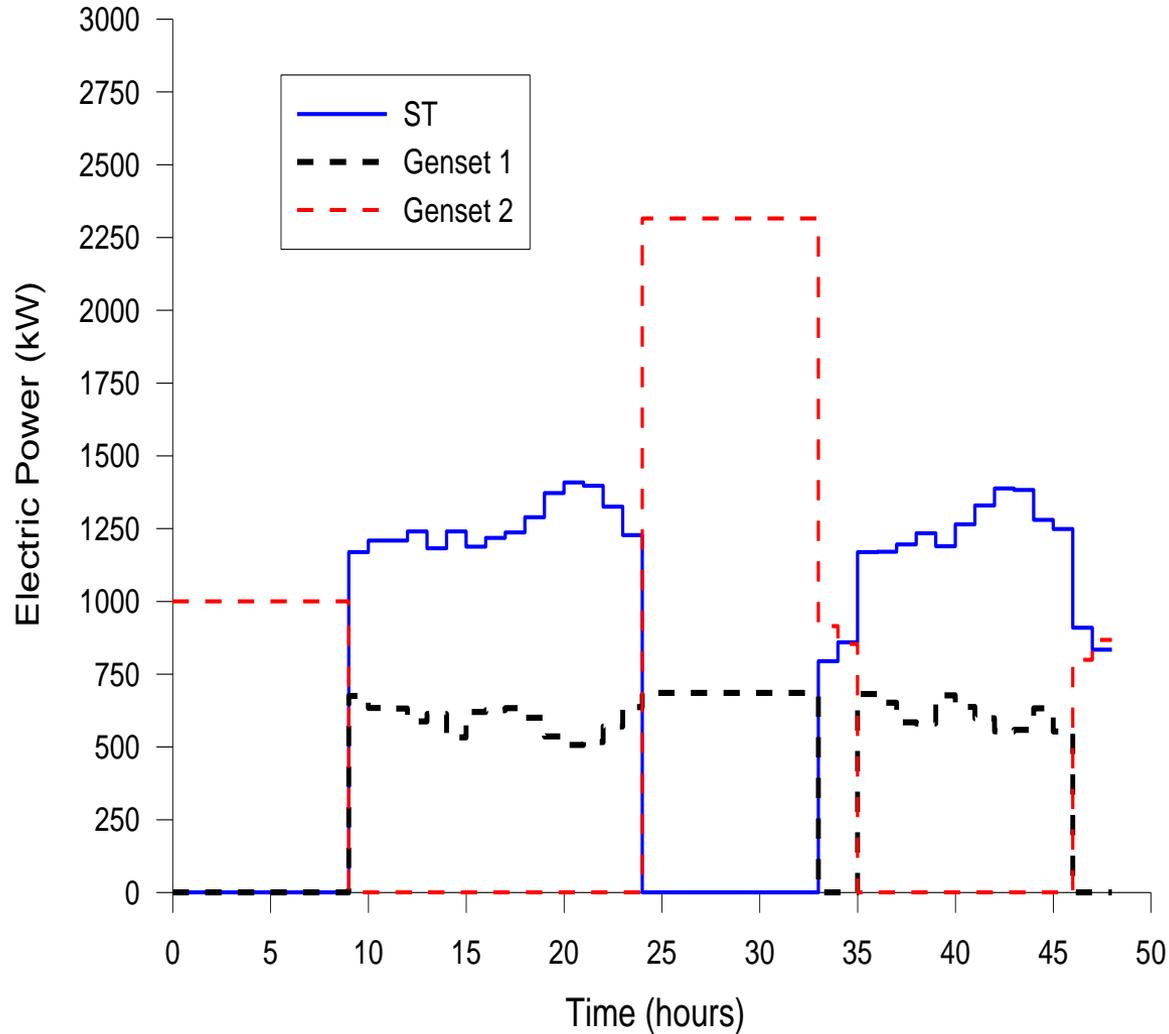
*Figure 5.6: Optimal load factors of the main engines versus time.*

## Example 2: Numerical Results



*Figure 5.7: Thermal power of the HRSG and the auxiliary boiler versus time.*

## Example 2: Numerical Results



**Figure 5.8: Electric power of the Diesel-generators and the steam turbine-generator versus time.**

### 5.2.5 Comments on the Results

- The optimal synthesis comprises two four-stroke Diesel engines of significantly different nominal power (14.84 MW and 6.15 MW) and two Diesel-generators also of different nominal power (715 kW and 2400 kW).
- Slow steaming through the storms is the result of operation optimization, in order to minimize the Diesel engine fuel cost.
- The load factor varies from 85% to 98%, for the smaller engine (area of minimum consumption in 4-X Diesel engines) and from 50% to 97% for the larger engine.
- The thermal demands during the trips are almost fully covered by the HRSG.

## Example 2: Comments on the Results

- The STG covers the  $\frac{2}{3}$  of the electric demand during the trips, while the remaining  $\frac{1}{3}$  is covered by the small Diesel-generator. For residual (not fully covered by the STG) loads smaller than 700 kW, only the smaller Diesel-generator operates, while for loads higher than 700 kW but lower than 2400 kW, only the large Diesel-generator operates. For higher demands, both Diesel-generators operate.
- The main engines efficiency is 46.48%, while the total efficiency of the system is 50.55%, which is higher than the main engines efficiency by 4.07%.

## 6. Closure

- The intertemporal static and intertemporal dynamic optimization of energy systems was the subject of this presentation, as it is applied for optimization of the system at three levels: synthesis (configuration), nominal characteristics (design specifications), and operation mode under various conditions.
- The concepts have been defined, the two types of problems have been stated mathematically and methods for their solution have been mentioned.
- Two example problems, one for static and one for dynamic optimization, help in clarifying the whole procedure and demonstrate the usefulness of applying optimization.

## 6. Closure

- **Optimization under transient conditions (e.g. load increase or decrease) is also dynamic optimization, but it is beyond the scope of this presentation. Here, it is assumed that transients take a small part of the whole life of a ship (and consequently of the energy system) and for this reason, they do not affect crucially the solution of the broad optimization problem, which is related to the whole life of a system (order of 20 years).**
- **“Conditions expected to be encountered”: Uncertainty for future states requires treatment with stochastic approaches.**

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# Thank you for your attention

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